NORTHERN BEACHES SECONDARY COLLEGE

MANLY SELECTIVE CAMPUS

HIGHER SCHOOL CERTIFICATE

Trial Examination

2018

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 3hours
- Use **black** pen
- Write your Student Number at the top of each page
- Section I – Multiple Choice – use the Answer Sheet provided
- Section II – Free Response – use a separate booklet for **each** question.
- NESA approved calculators and templates may be used.
- Reference sheet provided.

Section I Multiple Choice

- 10 marks
- Attempt all questions

Section II – Free Response

- Questions 11-16 – 90 marks
- Each question is of equal value
- All necessary working should be shown in every question.

Weighting: 40%

Section 1 Multiple Choice: Attempt Questions 1 – 10

Answer questions on the provided answer sheet.
Allow approximately 15 minutes for this section.

Q1 The point *M* is the midpoint of the points *A* and *B*. The coordinates of *A* are (2, -3) and the coordinates of *M* are (-2, 1). The coordinates of *B* are:

A (-6, 5)

B (5, -6)

C (0, -1)

D (-1, 0)

Q2 The correct solution to $|2x + 3| \geq 7$ is:

A $-2 \leq -x \leq 5$

B $x \leq -2, x \geq 5$

C $x \leq -5, x \geq 2$

D $-2 \leq x \leq 2$

Q3 The coordinates of the focus of the parabola $4y = (x + 2)^2 - 4$ is:

A (-2, 0)

B (-2, -1)

C (-2,-3)

D (2,-4)

Q4 If $\int_0^a (4-2x)\ dx = 4$, find the value of a.

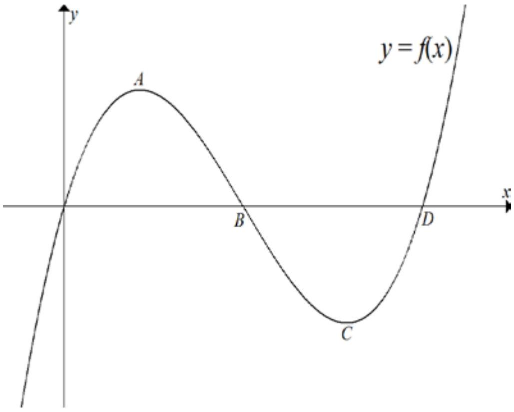
A $a = -2$

B $a = 0$

C $a = 4$

D $a = 2$

Q5 The diagram shows the graph of the function $y = f(x)$.



At which point is $f'(x) > 0$ and $f''(x) > 0$?

A A

B B

C C

D D

Q6 When the curve $y = e^x$ is rotated about the x – axis between x = -2 and x = 2, the volume of the solid generated is given by:

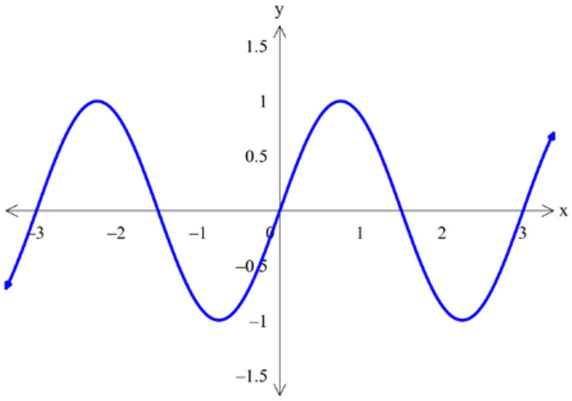
A $\pi\int_{-2}^{2} e^x \, dx$

B $2\pi\int_{0}^{2} e^{x^2} \, dx$

C $\pi\int_{-2}^{2} e^{x^2} \, dx$

D $\pi\int_{-2}^{2} e^{2x} \, dx$

Q7 In the diagram, the graph of the function $y = \sin(kx)$ is given.



Which could be the value of k?

A $\frac{3\pi}{2}$

B $\frac{2\pi}{3}$

C 3

D $\frac{2}{3}$

Q8 Which expression is a term of the geometric series $2x - 4x^3 + 8x^5 - \ldots$?

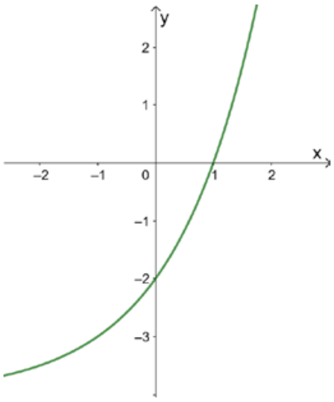
A $-2^{10}x^{19}$

B $2^{10}x^{19}$

C -2^9x^{19}

D 2^9x^{19}

Q9 In the diagram, the graph of the function $y = 2^{x+a} + b$ is given



Which could be the values of a and b?

A $a = 1$ and $b = 2$

B $a = 1$ and $b = -4$

C $a = 1$ and $b = -2$

D $a = -1$ and $b = 4$

Q10 The region bounded by the x-axis and the part of the graph $y = \cos x$ between $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$ is separated into two regions by the line $x = k$. If the area of the region for $-\frac{\pi}{2} \leq x \leq k$ is three times the area of the region for $k \leq x \leq \frac{\pi}{2}$ then the value of k equals ?

A $\sin^{-1}\frac{1}{4}$

B $\frac{\pi}{3}$

C $\frac{\pi}{4}$

D $\frac{\pi}{6}$

<u>End of Multiple Choice</u>

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.

In Questions 11–16, your responses should include relevant mathematical reasoning and/or calculations

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Question 11: Start A New Booklet 15 Marks

a. Factorise $12x^2 - 4x - 1$ **1**

b. Express $\dfrac{18}{3-\sqrt{3}}$ in the form $a + \sqrt{b}$ where *a* and *b* are integers. **2**

c. Differentiate $\dfrac{2x+1}{\sqrt{x}}$. **2**

d. Determine $\displaystyle\int_0^2 (5x-1)^9\,dx$ **3**

e. Determine the domain of the function $f(x) = \dfrac{x}{\sqrt{x^2-4}}$. **2**

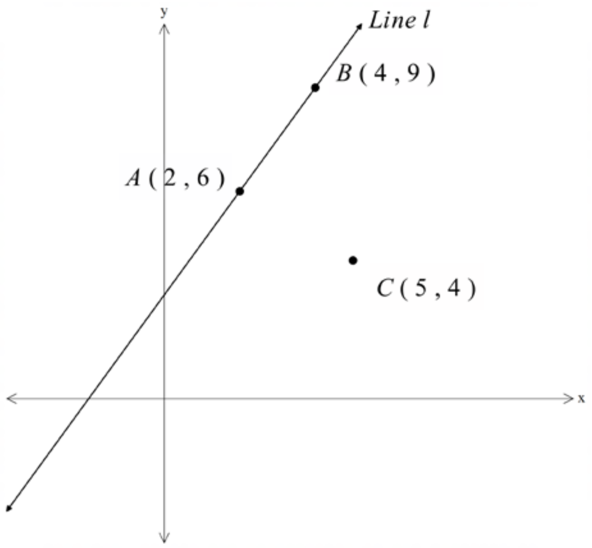
f. Given $\cos(2\theta) = \dfrac{1}{2}$ for $0 \leq \theta \leq \pi$, determine the value of $\sin\theta$? **2**

g. Evaluate $\displaystyle\int_0^2 \frac{6x}{x^2+2}\,dx$, leaving your answer in simplest exact form. **3**

End of Question 11

Question 12 Start A New Booklet 15 Marks

a. The line *l* connects the points A (2,6) and B (4,9).
Point C has the coordinates (5,4).



i. Determine the gradient of Line *l*. **1**

ii. Determine the exact length AB **1**

iii. Determine the area of triangle ABC **3**

iv. What angle, to the nearest degree, does the line through AC make with the positive x – axis? **2**

b. Given the simultaneous equations:

$$\begin{aligned} 2x + y &= 1 \\ x^2 - 4ky + 5k &= 0 \end{aligned}$$

where k is a non-zero constant

i. Show that $x^2 + 8kx + k = 0$ **1**

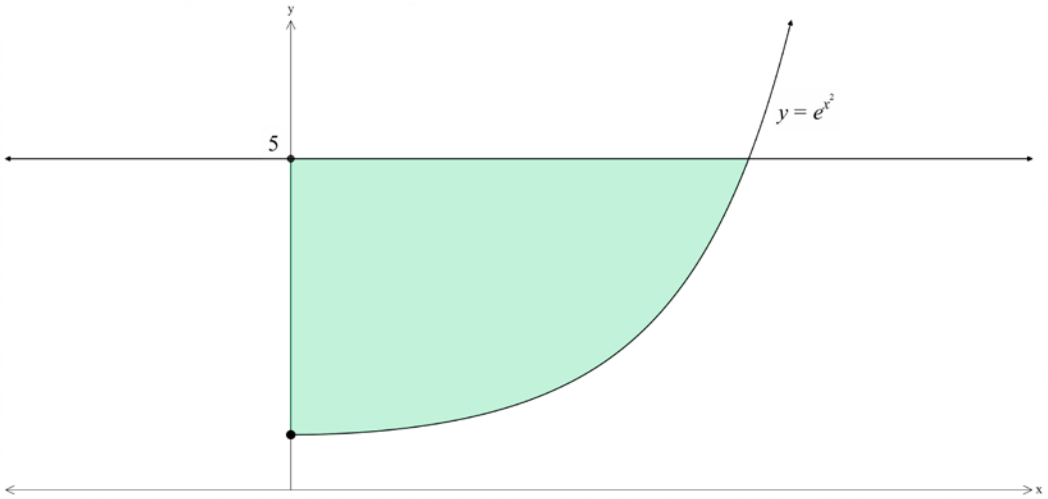
ii. Given that $x^2 + 8kx + k = 0$ has equal roots, find the value of k. **2**

iii. For this value of k, find the solution of the simultaneous equations. **1**

Question 12 continues on the next page.

Question 12 continued:

c. The shaded region bounded by the graph $y = e^{x^2}$, the line $y = 5$ and the y – axis is rotated about the y – axis to form a solid of revolution.



i. Show that the volume of the solid is given by

$$V = \pi\int_1^5 \ln y \; dy$$ **1**

ii. Use Simpsons Rule with five functions values to approximate the volume of the soliud of revolution V_y correct to three decimal places. **3**

End of Question 12

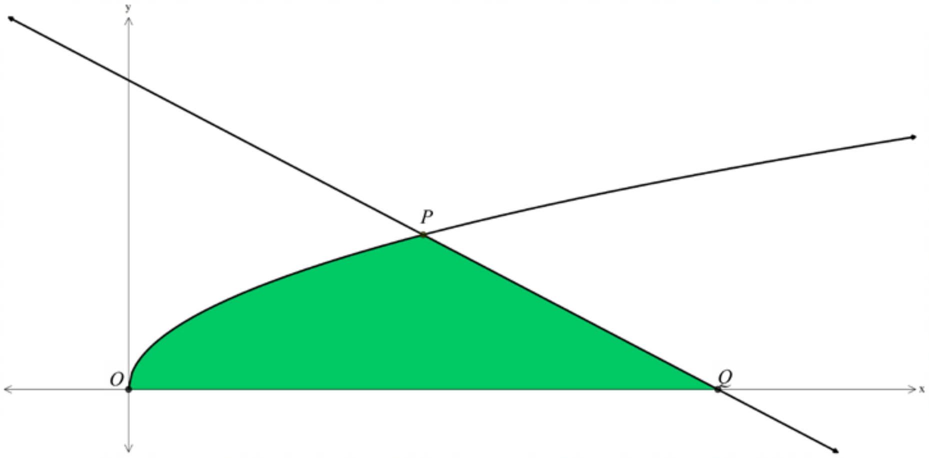
Question 13 Start A New Booklet 15 Marks

a. The derivative of a function is given by $f'(x) = 3x^2 + 3$. The curve passes through the point (1, 1). Find the equation of the curve. **2**

b. i. Find the sum of the sequence 100, 101, 102, ……, 999. **1**

ii. Hence, or otherwise, find the sum of all the 3 digit numbers which are not divisible by 5. **2**

c. In the diagram below, the shaded region is bound by graphs of $y = 8 - x$, $y = 2\sqrt{x}$ and the x – axis.



i Determine the coordinates of point P. **2**

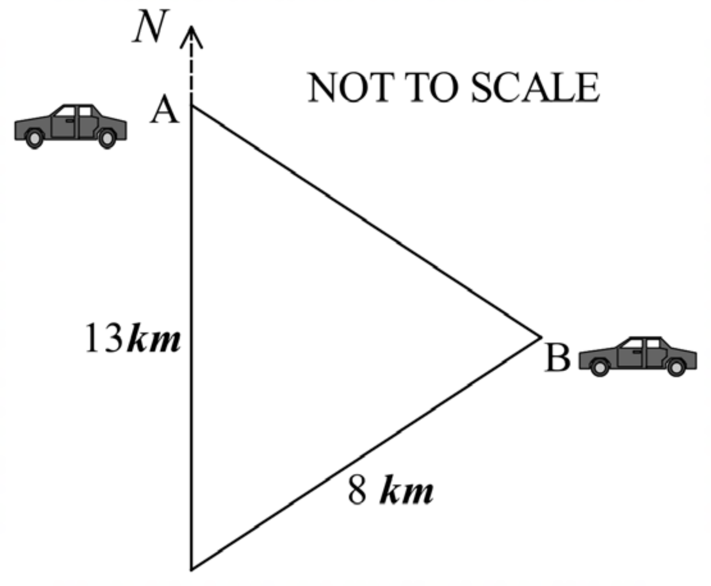
ii Find the exact value of the shaded region OQP. **2**

d. Find the equation of normal to the curve $y = \sqrt{x^2 + 7}$ at $x = 3$. **3**

Question 13 continues on the next page.

Question 13 continued:

e. Two vehicles, Car A and Car B, depart from the same starting position. Car A travels north for a distance of 13 km. At the same time Car B travels on a bearing 040° T for a distance of 8 km.



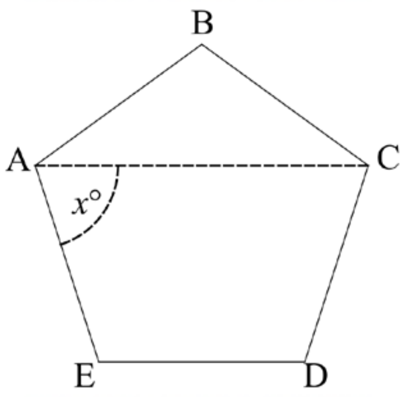
i. What is the distance between the cars at this time? **1**

ii. What is the bearing of Car A from Car B? **2**

End of Question 13

Question 14 Start A New Booklet 15 Marks

a. *ABCDE* is a regular pentagon and $\angle CAE = x°$



Determine the value of *x*, giving reasons for your answer. **2**

b. i. Differentiate $x \cos 2x$. **2**

ii. Hence find $\int_0^{\frac{\pi}{6}} x \sin 2x \, dx$ **2**

c. i. Sketch the curve of $y = |x - 2| - 5$ **2**

ii Hence solve $|x - 2| - 5 < 2x$ **1**

d. For the curve given by $f(x) = 2x^2 e^{-x}$

i. Find any stationary points and determine their nature **3**

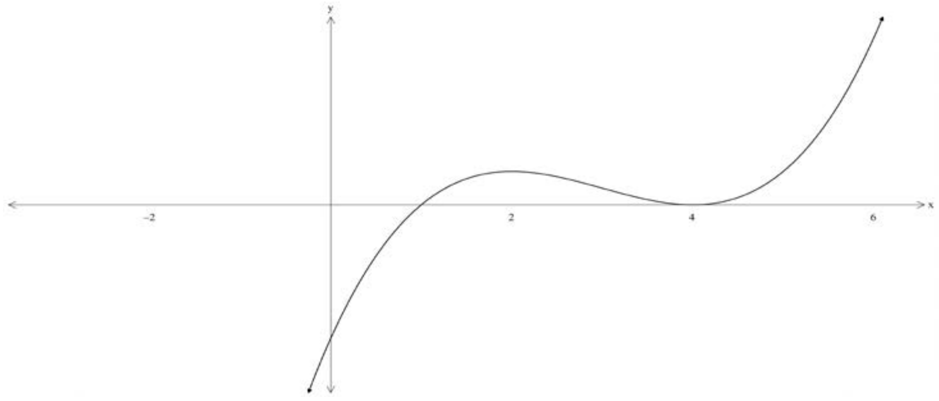
ii Determine $\lim_{x \to \infty} f(x)$ **1**

iii Hence sketch the curve **2**

End of Question 14

Question 15 Start A New Booklet 15 Marks

a. The graph of $y = f'(x)$ is given below.



Copy or trace the graph onto you answer booklet, using at least a third of the page.

Sketch the graph of $y = f(x)$ given that it passes through the points $(0,0)$ and $(4,-2)$.
Show clearly any turning points and/or points of inflexion. **3**

b. The velocity of a particle is given by:

$\dot{x} = 1 - 2\sin\pi t$, where x is in metres and t, is the time in seconds.

Initially the particle is $\frac{2}{\pi}$ m to the right of the origin.

i. Find the acceleration of the particle in terms of t. **1**

ii Find the values of t in the interval $0 \leq t \leq 4$ where the particle is at rest. **2**

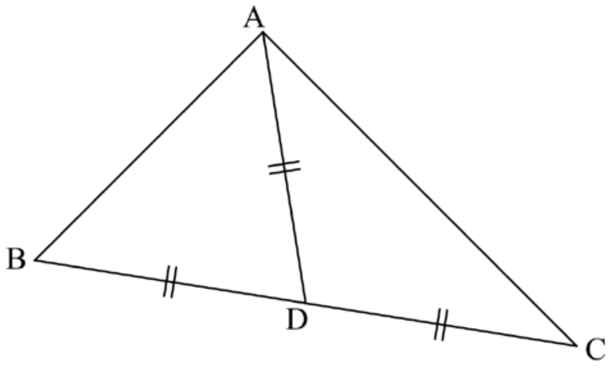
iii. Sketch the graph $\dot{x} = 1 - 2\sin\pi t$ as a function of time for $0 \leq t \leq 4$,
Show all intercepts on the horizontal and vertical axes. **3**

ii. Find the distance travelled in the interval $t = 1$ to $t = 2$. **2**

Question 15 continues on next page.

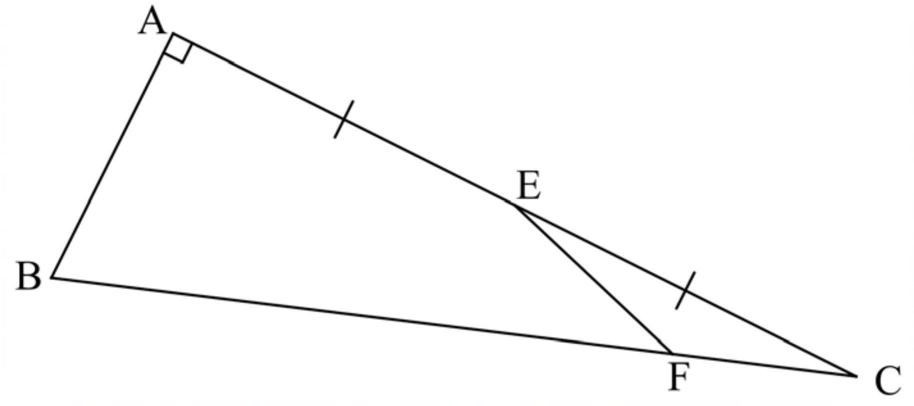
Question 15 continued:

c. In the triangle *ABC*, *AD* = *BD* = *CD*



i. Prove $\angle BAC = 90°$. **2**

ii. In triangle *ABC*, $\angle BAC = 90°$, *AE* = *CE* , *EF* = *8* and $\dfrac{BC}{FC} = 4$



Using part i , or otherwise, determine the length of *BC*. **2**

End of Question 15

Question 16 Start A New Booklet 15 Marks

a. A bacteria population, P, in a container is modelled by:

$$P(t) = 220 - Ae^{kt} \ ,$$

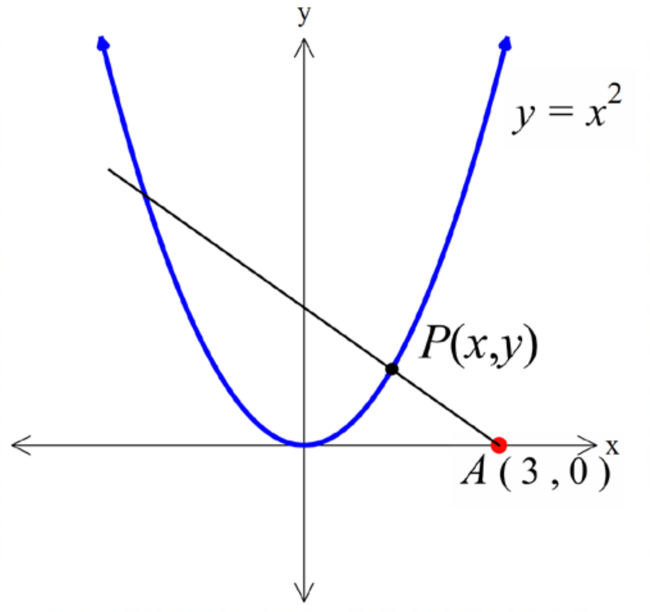
where t is in days and P expressed in thousands.
Initially there were 100 000 bacteria in the container.

i. State the value of A. **1**

ii. If the population reaches 180 000 in two days, find the exact value of k. **2**

iii. Find the limiting bacteria population in the container. **1**

b. The diagram shows the curve $y = x^2$ and the point P (x , y) and A (3, 0).



i. Show that $2(x-3) + 4x^3 = 2(x-1)(2x^2+2x+3)$ **1**

ii. Hence determine the coordinates of P such that the distance PA is a minimum. **4**

Question 16 continues on next page.

Question 16 continued:

c. Denise and Damien borrow $500 000 to start a business. The interest rate is 7.2% p.a. compounding monthly. They plan to repay the loan in 20 years with equal regular monthly repayments.

The amount they owe after the *n*-th repayment is given by:

$$A_n = Pr^n - M(1 + r + ... + r^{n-1})$$

$$\text{where } P = \$500\ 000\ ,\quad r = 1 + \frac{0.072}{12}$$

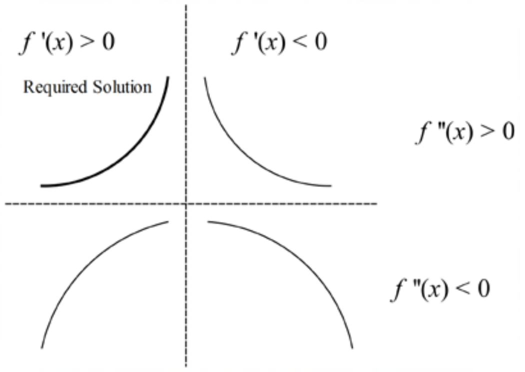
and *M* is the amount of the regular monthly payment.

i. Show the amount of their regular payment is $3936.75. **2**

ii. After 4 years of regular payments, Denise and Damien make an extra payment of $70 000. They then continue to make quarterly (once every three months) payments of $10 000. All other conditions remain the same.

Explain if they will pay off the loan earlier than they initially planned. Justify your answer with calculations. **4**

End of Examination

Multiple Choice		
Question	Answer	Solution
1	A	$x = \frac{x_1 + x_2}{2} \quad y = \frac{y_1 + y_2}{2}$ $-2 = \frac{2 + x}{2} \quad 1 = \frac{-3 + y}{2}$ $-4 = 2 + x \quad 2 = -3 + y$ $x = -6$ and $y = 5$ $\therefore A$
2	C	2 cases positive and negative $|2x + 3| \ge 7 \qquad -(2x + 3) \ge 7$ $2x + 3 \ge 7 \qquad -2x - 3 \ge 7$ $2x \ge 4 \quad -2x \ge 10$ $x \ge 2\, x \le -5$ $\therefore x \le -5, x \ge 2$ $\therefore C$
3	A	$(x + 2)^2 = 4y + 4$ $(x + 2)^2 = 4(y + 1)$
4	D	$A = \frac{a}{2}(4 + 4 - 2a)$ $= 4a - a^2$ $\therefore \quad 4 = 4a - a^2$ $(a - 2)^2 = 0$ $a = 2$
5	D	
6	D	$V = \pi \int_a^b y^2 dx$ $y^2 = (e^x)^2 = e^{2x}$ $V = \pi \int_{-2}^{2} e^{2x}\, dx$
7	B	Period: $\frac{2\pi}{k} = 3$ $k = \frac{2\pi}{3}$

<table>
<tr>
<td>8</td>
<td>A</td>
<td>
$2^1 x^1 - 2^2 x^3 + 2^3 x^5 - 2^4 x^7 + ...$
<table>
<tr><td>Power of x</td><td>1</td><td>3</td><td>5</td><td>7</td><td>9</td><td>…</td></tr>
<tr><td>Coefficient</td><td>2^1</td><td>-2^2</td><td>2^3</td><td>-2^4</td><td>2^5</td><td>…</td></tr>
</table>

Every 2nd term is negative, so the coefficient of x^{19} is negative.
The power of x in each term is twice the power of 2, minus 1. So, if the power of x is 19, then the power of 2 is 10.
$\therefore -2^{10} x^{19}$

Alternatively.

Geometric Series
$a = 2x \quad r = {}^{-}2x^2$

$T_n = ar^{n-1} = 2x({}^{-}2x^2)^{n-1}$

$= 2({}^{-}2)^{n-1} x \times x^{2n-2}$
$= 2({}^{-}2)^{n-1} x^{2n-1}$

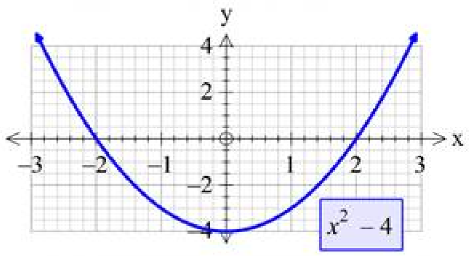
but each term in options is x^{19}
therefore $n = 10$

$T_n = 2 \times ({}^{-}2)^9 x^{19} = {}^{-}2^{10} x^{19}$
</td>
</tr>
<tr>
<td>9</td>
<td>B</td>
<td>
$-2 = 2^a + b \ (1)$
$0 = 2^{a+1} + b \ (2)$
$0 = 2.2^a + b$
$b = -2.2^a$
$-2 = -2.2^a + 2^a$
$2^a = 2$
$a = 1$
$b = -4$
</td>
</tr>
<tr>
<td>10</td>
<td>D</td>
<td>
$\int_{-\frac{\pi}{2}}^{k} \cos dx = 3\int_{k}^{\frac{\pi}{2}} \cos dx$

$\sin k - \sin(-\frac{\pi}{2}) = 3(\sin(\frac{\pi}{2}) - \sin k)$
$\sin k + 1 = 3 - 3\sin k$
$4\sin k = 2$
$\sin k = \frac{1}{2}$
$k = \frac{\pi}{6}$
</td>
</tr>
</table>

Q11

a	$12x^2 - 4x - 1$ $(6x+1)(2x-1)$	1 mark correct solution
b	$\frac{18}{3-\sqrt{3}}$ $= \frac{18}{3-\sqrt{3}} \times \frac{3+\sqrt{3}}{3+\sqrt{3}}$ $= 18 \times \frac{3+\sqrt{3}}{9-3}$ $= 18 \times \frac{3+\sqrt{3}}{6}$ $= 3(3+\sqrt{3})$ $= 9 + 3\sqrt{3}$ $= 9 + \sqrt{27}$ $\therefore\ a = 9$ and $b = 27$	2 marks correct solution 1 mark correct multiplication and expansion of denominator
c	$\frac{d}{dx}\frac{2x+1}{\sqrt{x}}$ $= \frac{d}{dx}\frac{2x+1}{x^{\frac{1}{2}}}$ $= \frac{d}{dx} 2x^{\frac{1}{2}} + x^{-\frac{1}{2}}$ $= x^{-\frac{1}{2}} - \frac{1}{2}x^{-\frac{3}{2}}$ Product rule and quotient rule also work but are much harder to apply	2 marks correct solution 1 mark for correct simplification of both parts prior to differentiation or if used product or quotient rule correct application of formula
d	$\int_0^2 (5x-1)^9$ $= \left[\frac{5x-1}{10 \times 5}\right]_0^2$ $= \frac{1}{50}\left((5 \times 2 - 1)^{10} - (-1)^{10}\right)$ $= \frac{9^{10} - 1}{50}$ $= 69\ 735\ 688$	3 marks correct solution 2 marks correct integration and correct substitution 1 mark correct integration but incorrect or no substitution. Answer in index form preferred

e	 $f(x) = \frac{x}{\sqrt{x^2 - 4}}$ Domain will exist if $x^2 - 4 > 0$ Only part of the curve that is above but not equal to points on x axis ∴ From graph above $x < -2$ and $x > 2$	2 marks correct solution 1 mark if only gives one part of solution or of gives = -2 or 2 as part of solution
f	$\cos(2\theta) = \frac{1}{2}$ for $0 \leq \theta \leq \pi$ $\beta = 2\theta$ for $0 \leq \beta \leq 2\pi$ $\cos(\beta) = \frac{1}{2}$ Using $ASTC$ ∴ Quad 1 and 4 $\beta = \frac{\pi}{3}$ and $\frac{5\pi}{3}$ ∴ $\theta = \frac{\pi}{6}$ and $\frac{5\pi}{6}$ ∴ $\sin\theta = \frac{1}{2}$	2 marks correct solution 1 mark if only finds angle with no value for sin
g	$\int_0^2 \left(\frac{6x}{x^2 + 2}\right) dx$ note:numerator is 3 times diff of denominator so log function use $\log_e = \ln$ $= 3 \times \int_0^2 \left(\frac{2x}{x^2 + 2}\right) dx$ $= 3 \times [\ln(x^2 + 2)]_0^2$ $= 3[\ln(6) - \ln(2)]$ $= 3\ln\frac{6}{2}$ $= 3\ln(3)$ or $\ln(27)$	3 marks correct solution 2 marks correct integration and substitution but not in simplest form 1 mark correct integration

Q12

ai	$m = \frac{9-6}{4-2}$ $= \frac{3}{2}$	1 mark correct solution
aii	$d = \sqrt{(4-2)^2 + (9-6)^2}$ $= \sqrt{13}$	1 mark correct solution
aiii	$y - 6 = \frac{3}{2}(x-2)$ $2y - 12 = 3x - 6$ $3x - 2y + 6 = 0$ $d \perp = \frac{\lvert 3(5) = (-2)(4) + 6 \rvert}{\sqrt{(3)^2 + (-2)^2}}$ $= \frac{13}{\sqrt{13}}$ $A = \frac{1}{2}(\sqrt{13})(\sqrt{13})$ $= \frac{13}{2} u^2$	3 marks correct solution 2 mark correct solution without proof for perp. 1 mark one correct step only either in perp distance, or gradient of AC or distance AC
aiv	$m_{\text{AC}} = \frac{4-6}{5-2}$ $= -\frac{2}{3}$ $\frac{2}{3} = \tan(180 - \theta)$ $\theta = 180 - \tan^{-1}\left(\frac{2}{3}\right)$ $= 180 - 34$ $= 146°$	2 mark correct solution 1 mark correct acute angle
bi	$y = 1 - 2x$ $x^2 - 4k(1-2x) + 5k = 0$ $x^2 - 4k + 8kx + 5k = 0$ $x^2 + 8kx + k = 0$	1 mark correct solution
bii	$\Delta = 0$ $\therefore (8\mathbf{k})^2 - 4\mathbf{k} = 0$ $64\mathbf{k}^2 - 4\mathbf{k} = 0$ $\mathbf{k} = 0 \text{ or } \mathbf{k} = \frac{1}{16}$ $\text{but } \mathbf{k} \neq 0 \therefore \mathbf{k} = \frac{1}{16}$	2 mark correct solution 1 mark correct solution but no rejection for k=0

<table>
<tr><td>biii</td><td>$x^2 + \frac{1}{2}x + \frac{1}{16} = 0$
$\left(x + \frac{1}{4}\right)^2 = 0$
$x = -\frac{1}{4}$</td><td>1 mark correct solution</td></tr>
<tr><td>ci</td><td>$y = e^{x^2}$
$x^2 = \ln y$ and $x = 0$, $y = 1$
$\therefore \quad V = \pi\int_1^5 x^2\,dy$
$= \int_1^5 \ln y\,dy$</td><td>1 mark correct solution</td></tr>
<tr><td>cii</td><td><table><tr><td>y</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>$\ln y$</td><td>$\ln 1$</td><td>$\ln 2$</td><td>$\ln 3$</td><td>$\ln 4$</td><td>$\ln 5$</td></tr></table>$V = \frac{\pi}{6}[(\ln 1 + \ln 5) + 4(\ln 2 + \ln 4) + 2(\ln 3)]$
$= 12.1244$
$= 12.124\ u^3$</td><td>3 marks correct solution
2 mark partial correct solution with one mistake only
1 mark correct values in table</td></tr>
</table>

COMMENTS

a) iii) many students found area without first showing right angled triangle
  iv) common mistake to state the acute angle despite diagram showing an obtuse angle

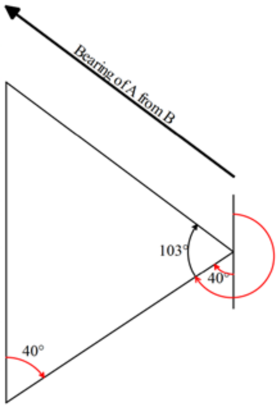
b) ii) divisions which resulted in the loss of the zero solution without reasons did not get full marks
c) i) lower bound needed to be shown by calculation as well as manipulation of expression
  ii) common mistake in omitting pi from calculations

Q 13

a	$f\ '(x)\ 3x^2 + 3$ $f(x) = \int 3x^2 + 3\ \ dx$ $= x^3 + 3x + c$ $x = 1 \ \Rightarrow\ y = 1$ $1 = 1 + 3 + c$ $c = {}^-3$ $f(x) = x^3 + 3x - 3$	2 marks – correct solution 1 mark – correct integral.
b-i	$Tn = a + (n - 1)d$ $999 = 100 + (n - 1) \times 1$ $n = 999 - 100 + 1 = 900$ $S_n = \frac{n}{2}(a + l)$ $= \frac{900}{2}(100 + 999)$ $= 494500$	1 mark – correct answer
b-ii	Sum of multiples of 5 from 100 to 995 $Tn = 100 + (n - 1) \times 5$ $995 = 100 + 5n - 5$ $n = \frac{900}{5} = 180$ $Sn = \frac{n}{2}(a + l)$ $= \frac{180}{2}(100 + 995)$ $= 98550$ $\therefore$ nonfactors of 5 $S_n = 494550 - 98550$ $= 396000$	2 marks – correct answer 1 mark – correct *n* - Correct answer from incorrect *n*

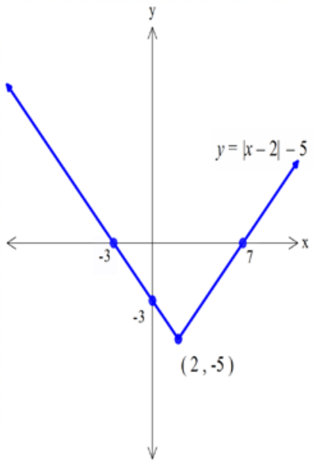
c-i	$8 - x = 2\sqrt{x}$ let $m = \sqrt{x}$ $8 - m^2 = 2m$ $m^2 + 2m - 8 = 0$ $(m + 4)(m - 2) = 0$ $m = {}^-4$ or $m = 2$ $\sqrt{x} \neq {}^-4$ ∴ $\sqrt{x} = 2$ $x = 4$ Coordinate $y = 8 - 4$ $P(4,4)$	2 marks – correct answer including explanation for why coordinate (4,4) was correct. 1 mark – $x = 4$ without further interpretation and/ or y - value
c-ii	Shaded Region $A = \int_0^4 2\sqrt{x}\ dx + \text{Area of Triangle}$ $= 2\left[\frac{2}{3}x^{\frac{3}{2}}\right]_0^4 + \frac{1}{2} \times 4 \times 4$ $= \frac{4}{3}\{(\sqrt{4})^3 - 0\} + 8$ $= \frac{32}{3} + 8 = \frac{56}{3}u^3$ Note: no marks awarded for $\int_0^8 2\sqrt{x} - (8 - x)\ dx$ or similar as this is not the defined shaded region.	2 marks – correct answer 1 mark - Either subregion correct using correct expression for total area

d	$y=\sqrt{x^2+7}$ $x=3 \;\Rightarrow\; y=\sqrt{9+7}=4$ (3,4) $\frac{dy}{dx}=2x\times\frac{1}{2}\times(x^2+7)^{-\frac{1}{2}}$ $=\frac{x}{\sqrt{x^2+7}}$ at $x = 3$ $m_{\text{Tangent}}=\frac{3}{\sqrt{16}}=\frac{3}{4}$ $\therefore \quad m_{\text{Normal}} = -\frac{4}{3}$ $y-4=-\frac{4}{3}(x-3)$ $3y-12={}^-4x+12$ $4x+3y-24=0$	3 marks correct solution 2 marks - Correct equation from incorrect gradient - Correct gradient for the normal. 1 mark Identifying - $m_T \times m_N = {}^-1$ - correct initial differentiation
e-i	$c^2=a^2+b^2-2ab\cos C$ $c=\sqrt{13^2+8^2-2\times 13\times 8\times \cos 40}$ $=8{\cdot}5827$ $\cong\ 8{\cdot}6\text{km}$	1 mark – correct solution .
e-ii	$\cos B=\frac{a^2+c^2-b^2}{2ac}$ $B=\cos^{-1}\left(\frac{8^2+8{\cdot}582^2-13^2}{2\times 8\times 8{\cdot}582}\right)$ $=103°11'$ If using Sine Rule – take care – 2 possible answers $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin(180-B)}{b}$ $\frac{\sin 40}{8{\cdot}5827}=\frac{\sin B}{13}$ $B=\sin^{-1}\left(\frac{13\times\sin 40}{8{\cdot}5827}\right)$ $=76°48' \text{ or } 103°\ 12°$ But 76^o does not fit with required dimensions.	2 mark – correct solution 1 mark - correct bearing from an incorrect use of Sine rule. - - Correct angle but incorrect bearing

	Bearing of A from B = 103 +40+180 = 323° 	

Q14

a	$\angle BAE = \frac{180(5-2)}{5}$ (angle sum of regular pentagon) $= 108°$ Similarly, $\angle B = 108°$ $AB = BC$ (regular pentagon) $\therefore \Delta ABC$ is isosceles $\angle BAC = \frac{180-108}{2}$ (base $\angle$s of isosceles Δ; angle sum of a Δ) $= 36$ $\therefore\ x = 108 - 36$ $= 72$	2 marks 1 mark deducted for unclear/insufficient reasoning/ just the correct value of *x*
bi	Let $u = x$ $\quad v = \cos 2x$ $u' = 1$ $\quad v' = -2\sin 2x$ $\frac{d}{dx}(x\cos 2x)$ $= uv' + vu'$ $= -2x\sin 2x + \cos 2x$	2 marks 1 mark for one error
bii	From (i): $\frac{d}{dx}(x\cos 2x) = -2x\sin 2x + \cos 2x$ $\therefore\ -2x\sin 2x = \frac{d}{dx}(x\cos 2x) - \cos 2x$ Now, $\int_0^{\frac{\pi}{6}} x\sin 2x\, dx$ $= -\frac{1}{2}\int_0^{\frac{\pi}{6}} -2x\sin 2x\, dx$ $= -\frac{1}{2}\int_0^{\frac{\pi}{6}} \left(\frac{d}{dx}(x\cos 2x) - \cos 2x\right) dx$ $= -\frac{1}{2}\left[x\cos 2x\right]_0^{\frac{\pi}{6}} + \frac{1}{2}\int_0^{\frac{\pi}{6}} \cos 2x\, dx$ $= -\frac{1}{2}\left[x\cos 2x\right]_0^{\frac{\pi}{6}} + \frac{1}{2}\left[\frac{1}{2}\sin 2x\right]_0^{\frac{\pi}{6}}$ $= -\frac{1}{2}\left(\frac{\pi}{6}\cos\frac{\pi}{3}\right) + \frac{1}{4}\sin\frac{\pi}{3}$ $= -\frac{\pi}{24} + \frac{\sqrt{3}}{8}$	2 marks 1 mark for a correct indefinite integral/ a correct expression using part (i) Note: Many students made errors with minus signs and fractions. Show all steps to avoid silly mistakes.
ci	Absolute value graph $y = \lvert x \rvert$ shifted right 2 units, down 5 units	2 marks



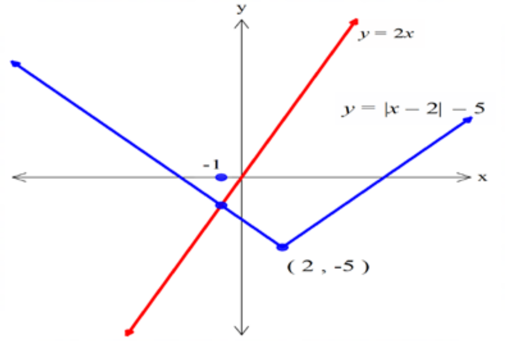
1 mark for a different absolute value graph with correct *x*-intercepts

Note: two points should be labelled on each branch (so that each branch is uniquely defined)

cii

Solve $|x-2|-5<2x$

Sketch *y*=LHS and *y*=RHS on the same set of axes, and see where the absolute value graph is underneath the line $y = 2x$



The left branch of the absolute value graph has equation:

$$y = -(x-2)-5$$
$$y = -x+2-5$$
$$y = -x-3$$

So for the point of intersection:

$$2x = -x-3$$
$$3x = -3$$
$$x = -1$$

$\therefore x > -1$ is the solution

1 mark correct answer

Note: many tried unsuccessfully to solve purely algebraically

di

$$f(x) = 2x^2\,e^{-x}$$
$$\text{Let } u = 2x^2 \qquad v = e^{-x}$$
$$\therefore \qquad u' = 4x \qquad v' = -e^{-x}$$

$$f'(x) = 4xe^{-x} - 2x^2\,e^{-x}$$
$$f'(x) = 2x\,e^{-x}(2-x)$$

stationary points:

$$2xe^{-x}(2-x) = 0$$
$$x(2-x) = 0 \text{ (dividing by } 2e^{-x})$$
$$\therefore \qquad x = 0 \text{ or } 2$$

y-coordinates:

$$f(0) = 0$$
$$f(2) = 8e^{-2}$$

x	-0.1	0	0.1	1.9	2	2.1
f '(*x*)	-	0	+	+	0	-

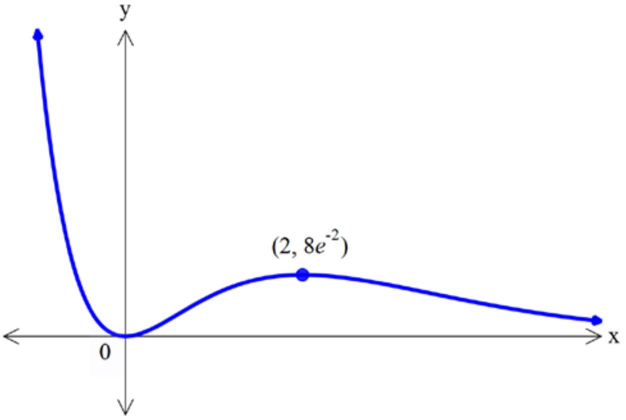
3 marks

2 marks for correctly finding and determining the nature of one stationary point

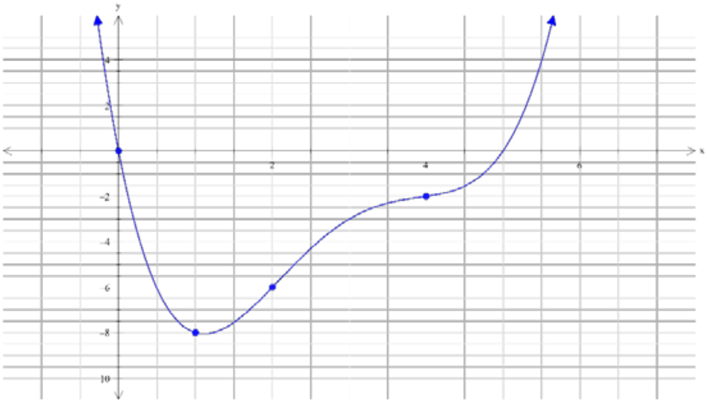
2 marks for finding both points but not determining their nature

1 mark for correct first derivative

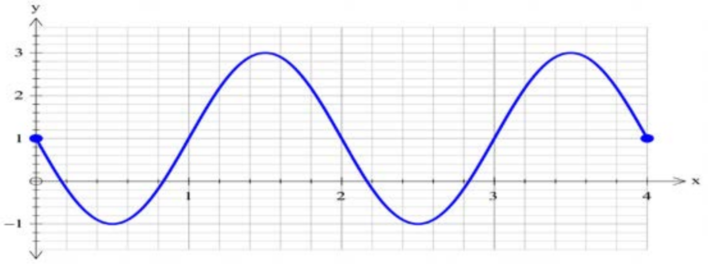
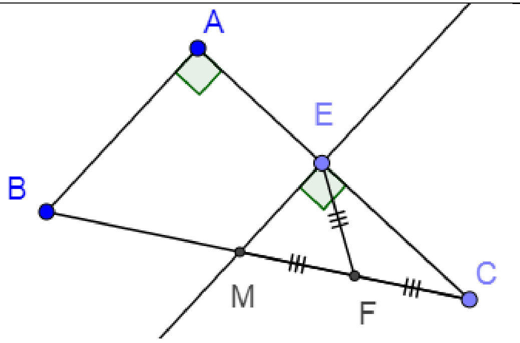
Note: Dividing both sides by *x* excludes the possibility that *x* = 0. When solving for *x*,

<table>
<tr>
<td></td>
<td>
<table>
<tr><td>Pictures showing sign of gradient</td><td>\</td><td>_</td><td>/</td><td>/</td><td>‾</td><td>\</td></tr>
</table>
Therefore, (0,0) is a minimum and (2, 8e^{-2}) is a maximum turning point.
</td>
<td>bring all terms to one side (= 0), then factorise and use the Null Factor Law.</td>
</tr>
<tr>
<td>dii</td>
<td>$\lim_{x \to \infty} 2x^2 e^{-x}$
$= \lim_{x \to \infty} \frac{2x^2}{e^x}$
$= 0$</td>
<td>1 mark correct answer</td>
</tr>
<tr>
<td>diii</td>
<td>
</td>
<td>2 marks for a graph showing all the results from d(i) and (ii)
1 mark deducted for a missing feature/ an incorrect feature (e.g. an extra stationary point)</td>
</tr>
</table>

Question 15

<table>
<tr>
<td>a)</td>
<td>

Horizontal point of inflection at (4,-2)
Minimum T.P. on line x=1 eg (1,-8)
Point of inflection on line x=2 eg (2,-6)</td>
<td>3 marks correct solution
2 marks 2 key points labelled of
Min TP POI HPOI
1 mark correct shape</td>
</tr>
<tr>
<td>bi)</td>
<td>$\ddot{x} = -2\pi \cos \pi t$</td>
<td>1 Correct answer</td>
</tr>
</table>

ii)	$\dot{x}=1-2\sin\pi t$ $\sin\pi t=\frac{1}{2}$ $\pi t=\frac{\pi}{6},\frac{5\pi}{6},\frac{13\pi}{6},\frac{17\pi}{6}$ $t=\frac{1}{6},\frac{5}{6},\frac{13}{6},\frac{17}{6}$	2 marks correct answer 1 mark 2 t values found
iii)	 P= 2 so 2 cycles range [-1,3] inverted raised 1 unit *x* ints from part ii)	3 marks correct answer 2 marks 1 mark correct correct y int and
iv)	$\int_1^2 1-2\sin\pi t\,dt=\left[t+\frac{2\cos\pi t}{\pi}\right]_1^2$ $=2+\frac{2\cos 2\pi}{\pi}-(1+\frac{2\cos\pi}{\pi})$ $=2+\frac{2}{\pi}-(1-\frac{2}{\pi})$ $=1+\frac{4}{\pi}\ or\ \frac{4+\pi}{\pi}$	2 marks correct solution 1 mark correct integration
ci)	In ΔABD Let $\angle DBA=\alpha$. Then $\angle BAD=\alpha$ (isosceles Δ) In ΔABD Let $\angle DAC=\beta$. Then $\angle DCA=\beta$ (isosceles Δ) Angle sum of $\Delta ABC=2\alpha+2\beta=180°$ $\alpha+\beta=90°=\angle BAC$ As required	2 marks correct solution 1 some progress
ii)	 Let $CF=x$, then $BC=4x$ Draw $EM \parallel AB$. From part (i): $EF=FM=CF=8=x\Rightarrow BC=4x=32$.	2 marks correct solution 1 mark CF=EF with reasons

Comments: Sketches using <u>scale</u> should be <u>smooth curves</u> and actually <u>resemble</u> the points featured.
b) SIN Sine Integrates Negative – double check with negatives c) proof format was mostly NOT adopted

Question 16

<table>
<tr><td>(a)(i)</td><td>$P(0)=100 \Rightarrow A=120$</td><td>1 mark: correct answer</td></tr>
<tr><td>(a)(ii)</td><td>$220-120e^{2k}=180 \Rightarrow e^{2k}=\frac{1}{3} \Rightarrow k=\frac{1}{2}\log_e \frac{1}{3}$</td><td>2 marks: correct answer
1 mark: progress towards solution</td></tr>
<tr><td>(a)(iii)</td><td>As $t \rightarrow \infty,\ -120e^{kt} \rightarrow 0,\ P(t) \rightarrow 220$
$\therefore 220\,000$</td><td>1 mark: correct answer (no mark for 220)</td></tr>
<tr><td rowspan="2">(b)(i)</td><td>$\text{RHS} = (2x-2)(2x^2+2x+3)$
$= 4x^3+4x^2+6x-4x^2-4x-6$
$= 2x-6+4x^3$
$= 2(x-3)+4x^3$
$= \text{LHS}$</td><td>1 mark: correct answer</td></tr>
<tr><td colspan="2">Marker's comments: Incorrect to treat as equation and solve or substituting an x-value to show both sides equal. Mark was awarded for subtracting expressions and finding equal to zero but this is not the normal way to show equivalence in expressions.</td></tr>
<tr><td>(b)(ii)</td><td>$\text{PA}^2 = (x-3)^2+y^2 = (x-3)^2+x^4$
let $s = (x-3)^2+x^4$
$\frac{ds}{dx} = 2(x-3)+4x^3 = 2(x-1)(2x^2+2x+3)$ from part (i)
stat pts at $\frac{ds}{dx}=0$
$x-1=0$ or $2x^2+2x+3=0$
$x=1$ No solution
at $x=1,\ y=1$
$\therefore P(1,1)$

Alternative method:
$\text{PA} = ((x-3)^2+x^4)^{\frac{1}{2}}$
$\text{PA}' = \frac{1}{2}((x-3)^2+x^4)^{-\frac{1}{2}}[2(x-3)+4x^3]$
$= \frac{2(x-3)+4x^3}{2\sqrt{(x-3)^2+x^4}}$
$= \frac{2(x-1)(2x^2+2x+3)}{2\sqrt{(x-3)^2+x^4}}$
stat pts at $\text{PA}'=0$
solution then follows as above

<table><tr><td>x</td><td>1^-</td><td>1</td><td>1^+</td></tr><tr><td>$\frac{ds}{dx}$</td><td>neg</td><td>0</td><td>pos</td></tr></table>Therefore minimum at P(1,1)</td><td>4 marks: correct answer
3 marks: correct solution without showing that the point is a minimum
2 marks: correct distance equation and derivative
1 mark for correct distance equation in terms of x

1 mark for correct distance equation
1 mark for differentiating and linking derivative with part (i)
1 mark for finding point
1 mark for showing PA is minimum</td></tr>
</table>

(c)(i)	Number of payments = $20\times 12=240$ $r = 1 + \frac{0.072}{12} = 1.006$ $(1 + r + ... + r^{n-1}) = \frac{r^{240} - 1}{r - 1}$ (sum of geometric series with $a = 1, n = 240$) $A_{240}=0 \Rightarrow Pr^{240} - M\frac{r^{240}-1}{r-1}=0$ Substituting *P* and *r,* $M=\frac{Pr^{240}\left(r-1\right)}{r^{240}-1}=\$3936.75$	2 marks: correct answer 1 mark: progress towards solution
(c)(ii)	Loan after extra payment (for $M=\$3936.75$): $A_{48}-70\,000=\$378071.26=Q$ Let B_n be the amount owing after the first payment of new schedule, $N=\$10000$ and $s=r^3$. $B_n=Qs^n-N\left(1+s+s^2+...+s^{n-1}\right)=Qs^n-N\frac{s^n-1}{s-1}$ $B_n=0\Rightarrow sQs^n-Qs^n-Ns^n+N=0$ $s^n=\frac{-N}{sQ-Q-N}\Rightarrow n=\log_s\left(\frac{-N}{sQ-Q-N}\right)$ Substituting *s*, *Q* and *N*, $n=64.3$ Since payments are made quarterly, loan will be paid off in 16.08 years. This is longer than the remaining term, 16 years. Alternative methods: 1) Substituting n=64 (number of quarters in 16 years) into B_n to find that $B_{64} = 2983.92$ (loan amount remaining after 20 years) 2) Substituting n=64 and $B_n = 378071.26$ to find $N \approx 10025.09$	4 marks: correct answer 3 marks: correct number of payments 2 marks: correct formula for amount owing in new schedule 1 mark: correct loan after extra payment

Marker's comments:
The question did not require showing how the formula A_n was derived. Students were able to use it directly, yet many still found A_1, A_2 etc (in both part (i) and (ii)

Common error in part (ii) was using the incorrect rate. Students who interpreted the question as the interest being compounded quarterly used $r = 1 + \frac{0.072}{4} = 1.018$ instead of $r = 1.006^3 \approx 1.018108216$